Models and Methods for
Random Networks: Exercise Set 1

**Exercise 1** Let $T$ be a random tree, where the number of children of vertices are i.i.d. random variables. Namely, each vertex at layer $n$ has a random number $D$ of children at layer $(n+1)$, independently of the number of children of other vertices. Suppose that $D$ can take 2 values $\{0, 2\}$, with $P(D = 0) = 1 - q$ and $P(D = 2) = q$ for some $0 < q < 1$. We define an i.i.d. bond percolation process, with open edge probability $p$ on this tree.

1. Compute the percolation probability $\theta(p)$.
2. Compute the percolation threshold $p_c$.

**Exercise 2** Let $T^2$ be a binary tree. We define the following i.i.d. site percolation process: a vertex is declared open with probability $p$, and closed with probability $(1 - p)$, independently for each vertex. When a vertex at layer $n$ is open, the two edges that connect it two its two children at layer $(n+1)$ are both open. When a vertex at layer $n$ is closed, the two edges that connect it two its two children at layer $(n+1)$ are both closed.

1. Compute the percolation probability $\theta(p) = P_p(|C| = \infty)$.
2. Compute the percolation threshold $p_c$.
3. Compute the mean cluster size $\chi(p)$.
4. Compare the three results obtained above with the values of i.i.d. bond percolation on the binary tree seen in class. Which ones are identical? How can some quantities be identical, whereas others are different? Explain briefly.