STOCHASTIC ANALYSIS OF REAL AND VIRTUAL STORAGE IN THE SMART GRID

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1. INTRODUCTION
Renewable but non dispatchable

Wind and PV require some mechanisms to compensate non dispatchability

Renewable Methods to Compensate for Fluctuations of PV and Wind

Dispatchable renewables

Storage
Demand Response
2. A MODEL OF DEMAND RESPONSE

Le Boudec, Tomozei, *Satisfiability of Elastic Demand in the Smart Grid*, Energy 2011 and ArXiv.1011.5606
Demand Response

- distribution network operator may interrupt / modulate power
- elastic loads support graceful degradation
- Thermal load (Voltalis), washing machines (Romande Energie«commande centralisée»)
e-cars

Voltalis Bluepod switches off thermal load for 60 mn
**Issue with Demand Response: Grid Changes Load**

- Widespread demand response may make load hard to predict

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**Intention**

- load with demand response
- «natural» load
- renewables

**Real**
Our Problem Statement

- Does demand response work?  
  - Delays  
  - Returning load

- **Problem Statement**  
  Is there a control mechanism that can stabilize demand?

- We make a macroscopic model of a transmission grid with large penetration of  
  - demand response  
  - Non dispatchable renewables

- We leave out for now the details of signals and algorithms
Starting Point: Macroscopic Model of Cho and Meyn [1], without Demand Response

Step 1: Day-ahead market

- Forecast demand: $D^f(t)$
- Forecast supply: $G^f(t) = D^f(t) + r_0$

Step 2: Real-time market

- Actual demand
  
  $D^a(t) = D(t) + D^f(t)$

- Actual supply
  
  $G^a(t) = G(t - 1) + G^f(t) + M(t)$

- Control (real time adjustment of Generation)

- Deterministic

- Random (deviation from forecast)
We add demand response to the model

- We capture two effects of Demand Response
  - Some load is delayed
  - Returning load is modified

- We do not model the IT aspects
  - Operation of Demand response is instantaneous
    (but has delayed impact)
Our Macroscopic Model with Demand Response

Ramping Constraint
\[-\xi \leq G(t) - G(t - 1) \leq \xi\]

Natural Demand
\[D^a(t) = D^f(t) + D(t)\]

Evaporation
\[\mu Z(t)\]

Returning Demand
\[B(t) = \lambda Z(t)\]

Backlogged Demand
\[Z(t)\]

Expressed Demand
\[E^a(t)\]

Frustrated Demand
\[F(t) = [E^a(t) - G^a(t)]^+\]

Satisfied Demand
\[\min(E^a(t), G^a(t))\]

Ramping Constraint
\[\min(G^a(t), G^a(t) + G^f(t) + M(t))\]

Control

Reserve (Excess supply)
\[R(t) = G^a(t) - E^a(t)\]
Demand that was subject to demand response is later re-submitted

- Delay term
  \[ \lambda Z \, dt \]
  \[ \frac{1}{\lambda} \text{ (time slots)} \] is the average delay

- Update term (evaporation):
  \[ \mu Z \, dt \]
  with \( \mu > 0 \) or \( \mu < 0 \)
  \( \mu \) is the evaporation rate (proportion of lost demand per time slot)
Assumption: \((M - D) = \text{ARIMA}(0, 1, 0)\)
typical for deviation from forecast
\[(M(t + 1) - D(t + 1)) - (M(t) - D(t)) = N(t + 1)\]
\[
\sim \text{iid with some finite variance}
\]
We obtain a 2-d Markov chain on continuous state space.

\[ -\xi \leq G(t) - G(t - 1) \leq \xi \]

\[ D^a(t) = D^f(t) + D(t) \]

\[ G^a(t) = G(t - 1) + G^f(t) + M(t) \]

\[ F(t) = [E^a(t) - G^a(t)]^+ \]

\[ R(t) = G(t - 1) - \lambda Z(t) + M(t) - D(t) + r_0 \]

\[ Z(t) = Z(t - 1) - \lambda Z(t) - \mu Z(t) + 1_{\{R(t) < 0\}} |R(t)| \]
The Control Problem

- Control variable: 
  \[ G(t - 1) \]
  production bought one time slot ago in real time market

- Controller sees only supply \( G^a(t) \) and expressed demand \( E^a(t) \)

- Our Problem: 
  keep backlog \( Z(t) \) stable

- Ramp-up and ramp-down constraints
  \[ \xi \leq G(t) - G(t - 1) \leq \zeta \]
Threshold Based Policies

\[ G^f(t) = D^f(t) + r_0 \]

\[ R(t) = G^a(t) - E^a(t) \]

Forecast supply is adjusted to forecast demand

\[ R(t) := \text{reserve} = \text{excess of demand over supply} \]

**Threshold policy:**

if \( R(t) < r \) * increase supply to come as close to \( r^* \) as possible (considering ramp up constraint)

else decrease supply to come as close to \( r^* \) as possible (considering ramp down constraint)
Simulations (evaporation $\mu > 0$)
Simulations (evaporation $\mu > 0$)

- $\mu > 0$ means returning load is, in average, less
- Large excursions into negative reserve and large backlogs are typical and occur at random times
Large backlogs may occur within a day, at any time (when evaporation $\mu > 0$)

Typical delay $\frac{1}{\lambda} = 30$ mn, all simulations with same parameters as previous slide, $\sigma = 160$
ODE Approximation \((\mu > 0)\) explain large excursions into positive backlogs
Simulations (evaporation $\mu < 0$)
Simulations (evaporation $\mu < 0$)

- $\mu < 0$ means returning load is, in average, more
- Backlog grows more rapidly

$\xi = \zeta = 100, \mu = -0.15 r^* = 300$ 1 time step = 10mn
ODE Approximation ($\mu < 0$) shows backlog is unstable
Findings: Stability Results

- If evaporation $\mu$ is positive, system is stable (ergodic, positive recurrent Markov chain) for any threshold $r^*$.
- If evaporation $\mu$ is negative, system unstable for any threshold $r^*$.
- Delay does not play a role in stability.
- Nor do ramp-up / ramp down constraints or size of reserve.
Evaporation

Negative evaporation $\mu$ means: delaying a load makes the returning load larger than the original one.

Could this happen?

Q. Does letting your house cool down now imply spending more heat in total compared to keeping temperature constant?

$\neq$ return of the load:

Q. Does letting your house cool down now imply spending more heat later?

A. Yes (you will need to heat up your house later -- delayed load)
Assume the house model of [6]

\[
\frac{d(t)\epsilon}{\text{heat provided to building}} = K(T(t) - \theta(t)) + C(T(t) - T(t-1))
\]

\[\text{leakiness outside inertia}\]
efficiency $\epsilon \sum_{t=1}^{\tau} d(t) = K \sum_{t=1}^{\tau} \left( T(t) - \theta(t) \right) + C(T(\tau) - T(0))$

E, total energy provided

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Optimal</th>
<th>Frustrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building temperature</td>
<td>$T^*(t), t = 0 \ldots \tau$</td>
<td>$T(t), t = 0 \ldots \tau, T(t) \leq T^*(t)$</td>
</tr>
<tr>
<td>Heat provided</td>
<td>$E^* = \frac{1}{\epsilon} \left( K \sum_{t=1}^{\tau} \left( T^<em>(t) - \theta(t) \right) + C(T^</em>(\tau) - T^*(0)) \right)$</td>
<td>$E &lt; E^*$</td>
</tr>
</tbody>
</table>

$\text{Scenario Optimal Frustrated}$

$\text{Building temperature}$

$\text{Heat provided}$

$\text{Efficiency}$

$\text{Total energy provided}$

$\text{Achieved} \ t^0$
Q. Does letting your house cool down now imply spending more heat in total compared to keeping temperature constant?

A. No, less heat
Findings

- Resistive heating system: evaporation is positive. This is why Voltalis bluepod is accepted by users.

- If heat = heat pump, coefficient of performance $\epsilon$ may be variable negative; evaporation is possible.

- Electric vehicle: delayed charge may have to be faster, less efficient, negative evaporation is possible.
What this suggests about Demand Response:

- Negative evaporation makes system unstable. Existing demand-response positive experience (with Voltalis/PeakSaver) might not carry over to other loads.

- Model suggests that large backlogs are possible and unpredictable.

- Backlogged load is a new threat to grid operation. Need to measure and forecast backlogged load.
3.

USING STORAGE TO COPE WITH WIND VOLATILITY

Gast, Tomozei, Le Boudec. Optimal Storage Policies with Wind Forecast Uncertainties, GreenMetrics 2012
Storage

- Stationary batteries, pump hydro

Cycle efficiency
\[ \approx 70 - 80\% \]
Operating a Grid with Storage

1a. Forecast load $D^f_t (t + n)$ and renewable supply $W^f_t (t + n)$
1b. Schedule dispatchable production $W^f_t (t + n)$

2. Compensate deviations from forecast by charging / discharging $\Delta$ from storage
Full compensation of fluctuations by storage may not be possible due to power / energy capacity constraints

- Fast ramping energy source ($CO_2$ rich) is used when storage is not enough to compensate fluctuation

- Energy may be wasted when
  - Storage is full
  - Unnecessary storage (cycling efficiency $< 100\%$)

- Control problem: compute dispatched power schedule $P^f_t (t + n)$ to minimize energy waste and use of fast ramping
Example: Wind data & forecasting

- Aggregate data from UK  
  (BMRA data archive [https://www.elexonportal.co.uk/](https://www.elexonportal.co.uk/))

- Demand perfectly predicted
- 3 years data
- Scale wind production to 20% (max 26GW)

\[ W(t) := \frac{\text{production}(t)}{\text{total wind capacity at time } t} \times 26\text{GW}. \]

- Relative error

\[ \frac{\sum_t |W_t^f(t + n) - W(t + n)|}{\sum_t W(t)} \]

- Day ahead forecast = 24%
- Corrected day ahead forecast = 19%
Example: The Fixed Reserve Policy

- Set $P_t^f (t + n)$ to $D_t^f (t + n) - W_t^f (t + n) + r^*$ where $r^*$ is fixed (positive or negative).

- Metric: Fast-ramping energy used (x-axis)
  Lost energy (y-axis) = wind spill + storage inefficiencies

Efficiency $\eta = 0.8$

Efficiency $\eta = 1$
A lower bound

Theorem. Assume that the error \( e(t+n) = W(t+n) - W_t^f(t+n) \) conditioned to \( \mathcal{F}_t \) is distributed as \( \mathcal{E} \). Then:

\[
(i) \quad \tilde{G} \geq \mathbb{E}[(\varepsilon + \tilde{u})^-] - \text{ramp}(\tilde{u}) \\
\bar{L} \geq \mathbb{E}[(\varepsilon + \tilde{u})^+] - \text{ramp}(\tilde{u})
\]

where \( \text{ramp}(\tilde{u}) := \mathbb{E}[\min(\eta(\varepsilon + \tilde{u})^+, \eta C_{\text{max}}, (\varepsilon + \tilde{u})^-, D_{\text{max}})] \)

(ii) The lower bound is achieved by the Fixed Reserve when storage capacity is infinite.

- Depends on storage characteristics
  - Efficiency, maximum power (but not on size)
- Assumption valid if prediction is best possible
Lower bound is attained for $B_{\text{max}} = 100 \text{GWh}$

\[ C_{\text{max}} = D_{\text{max}} = 2 \text{GW} \]

\[ C_{\text{max}} = D_{\text{max}} = 6 \text{GW} \]

Efficiency $\eta = 0.8$

Efficiency $\eta = 1$
The BGK policy [Bejan, Gibbens, Kelly 2012]

- Aims at keeping a constant level of stored energy

\[
W_t^f (t + n) = P_t^f (t + n) - D_t^f (t + n)
\]

- It is moderately sub-optimal for large energy storage capacity.
Small energy storage capacity?

BGK is far from lower bound – can one do better?

\[ B_{\text{max}} = 5 \text{GWh}, \quad C_{\text{max}} = D_{\text{max}} = 2 \text{GW} \quad \eta = 0.8 \]
Scheduling Policies for Small Storage

- Fixed Reserve: $u = r^*$
- BGK: compute $u$ so as to let storage level be close to nominal value $\lambda$
- Dynamic Reserve: compute $u$ so as to minimize average anticipated cost
  - Solved using an MDP model and policy iteration
Dynamic Reserve uses a Control Law

- Effective algorithm to the Dynamic Reserve policy

\[ B_{\text{max}} = 5 \text{GWh}, C_{\text{max}} = D_{\text{max}} = 2 \text{GW} \]

\[ B_{\text{max}} = 50 \text{GWh}, C_{\text{max}} = D_{\text{max}} = 6 \text{GW} \]

Efficiency \( \eta \) = 0.8

Efficiency \( \eta \) = 1
The Dynamic Reserve policies outperform BGK

- Trying to maintain a fixed level of storage is not optimal

![Graphs showing efficiency comparison between Dynamic Reserve and BGK policies](image)

Efficiency $\eta = 0.8$

Efficiency $\eta = 1$

BGK: maintain fixed storage lvl

Fixed Reserve

Dynamic reserve

Lower bound
What this suggests about Storage

- (BGK policy: ) Maintain storage at fixed level: not optimal
  - Worse for low capacity
  - There exist better heuristics

- **Lower bound** (valid for any type of policy)
  - depends on $\eta$ and maximum power
  - **Tight** for large capacity (>50GWh)
  - Still gap for small capacity

- 50GWh and 6GW is enough for 26GW of wind

- Quality of prediction matters
Conclusion: Demand Response vs Storage

Demand Response
- Attractive (little capital investment)
- Unpredictable effects

Storage
- Capital investment
- Can be managed and understood
Questions?


