Introduction to Demand Response
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Contents

1. What is demand response?
   An illustration with seven examples
   A taxonomy

2. Elements of theory
WHAT IS DEMAND RESPONSE?

Terminology
Demand Response (DR)
≈ Demand Side Management (DSM)

- **Demand Side Management**
  - electric utility manipulates user appliance
- **Demand Response**
  - Demand Side Management as a response to price
- in practice both phrases often used interchangeably
- ≥ 100 years old ("Load Management", inband tones "ripple control", AM signal)

A clothes dryer connected to a load control "smart" switch (Wikimedia Commons)
Demand Response (DR) = Demand Side Management (DSM)

Why invented?

1. To reduce costs for consumers
2. To save energy
3. To optimize management of the electrical grid
4. To prevent night operation of noisy equipment
5. Je ne sais pas

Why invented?

- electrical systems must balance energy instantly
- energy balance in electrical grid is mainly done by adjusting supply to demand:
  - scheduling and forecasting + large scale interconnection; frequency response; reserves
- demand response = adjust demand to supply is one of the tools used to manage the power grid
- energy efficiency is obtained by managing demand efficiently but is outside the scope of this tutorial
Examples of Use of Demand Response

- peak shaving

- response to failures (avoid blackout)
- mitigate volatility of wind and solar energy
- mitigate network problems (congestion, voltage)

What can be subject to Demand Response?

- Demand response applies to *elastic* loads (load = consumer of electricity)
- Non elastic loads
  - lighting, watching TV, hair drying

- Elastic loads
  - boiler, car or bicycle battery, data center, fridges and freezers, air conditioner, washing machine
Demand Response Example 1
Norway’s pilot study [Saele and Grande 2011]

- tariff is increased at pre-defined times (8-10, 17-19)
- users made aware of high tariffs and times
- In some homes heating is also directly controlled
- study concludes that it works

Norway’s pilot study [Saele and Grande 2011]
Demand Response may reduce prices

- 120 EUR/MWh difference between 2 areas inside Norway
- [Saele and Grande 2011] claims that the price peak would be suppressed with demand response
A similar example (GulfPower, USA)

[Borenstein et al 2002] 7/17/02 1-Hour Critical
(139 Homes)

Figure 3-h. Average Load and Load Reduction in Gulf Power CPP program. The TOU rate (11 a.m. to 8 p.m.) was 9.3 c/kWh. The 1- and 2-hour CPP was 29 c/kWh, an extra 20 c/kWh. The 1-hour CPP dispatch was at hour 17.

Source: Brian White, Gulf Power

Example 2: Romande Energie

- **Time of Use** tariff
  Night tariff is lower

- **Interruptible Supply**:
  interruptible supply
  (service is available e.g. 20 hours per day)
  [Le Boudec and Tomozei 2011]

http://www.romande-energie.ch/images/File/Tarifs/2013_tarifs_RE.pdf
Example 3: Voltalis

- Widely deployed in France
- **Interruptible Load**
  Voltalis device stops electrical resistive heating / boiler for at most 60 mn per day
- Device («Bluepod») receives GSM signal and stops thermal loads
- No charge / no payment
- Acceptance based on
  - Voltalis claims energy usage reduction
  - Good citizens
- Similar schemes with incentive payment to users: PeakSaver (Canada), www.pge.com (USA), New Zealand, NGT frequency service (UK)

Voltalis does not pay nor charge anything to consumers but claims that consumers benefit by seeing a reduced electricity bill. Do you think this is true?

1. Yes, there must be a reduction in total energy consumed
2. No, there cannot be any reduction in total energy consumed
3. Total energy consumed is increased
4. Ich weiss nicht
Example 4: Dynamic Demand

- also called frequency service
- smart fridges, smart boilers, smart heaters / HVACs
- recall that frequency is the first signal of power imbalance

- primary frequency control traditionally done with dynamic generators -- fossil fuel generators, using droop control

**Example 4: Dynamic Demand**

- **dynamic demand** is an alternative to dynamic generators
- How it works ("grid friendly controller")
  (underfrequency): fridge delays compressor when frequency drops and anticipates when freq. increases

[Molina-Garcia et al 2011]

[Mario Paolone]
Is something missing with this algorithm?

1. Nothing
2. Timers need to be randomized
3. Internal temperature needs to be taken into account
4. Outside temperature needs to be taken into account
5. Non lo so

Avoid synchronized response ⇒ [Molina-Garcia et al 2011] uses randomized Tdelay

Internal temperature should be accounted for
--- See [Christakou et al 2012] for a variant that accounts for internal temperature
Dynamic Demand

- Simulation results for [Molina-Garcia et al. 2011] with 10% of loads implementing dynamic demand in a hypothetical country grid:

  - Dynamic Demand $\approx$ doubles the reserve

- Fridges as primary/secondary response could provide ca 1 GW of reserve to UK grid [Milborrow 2009]

- 70% of secondary regulation power (8 sec to 3 mn) in the US can be provided by building air conditioning and heating fans alone [Hao et al. 2012]
Example 5: Boilers as Tertiary Reserve
[Sundstrom et al 2012]

- Primary reserve = real time
- Secondary reserve = within minutes
- Tertiary reserve = starts after 15 mn
- Thermal loads can be anticipated or delayed
- Upper and lower energy curves for one boiler give bounds on feasible energy provision *schedules*

![Diagram showing flexibility of a sample boiler with 6 kW equivalent energy storage at an initial energy level of 1.2 kWh, and an average consumption of 200W.]

**Boilers as Tertiary Reserve**

- Assume operator (“Service aggregator”) controls a large set of boilers and can predict the upper and lower bounds for the aggregate energy curves.

  Service aggregator can select a middle trajectory and therefore obtain some reserve that can be sold to grid.

  Can be implemented with pricing and /or smart meters

- Upper bound: deliver 3kW for 1.71h then 200 W
- Lower bound: deliver 0W for 6h then 200 W

![Diagram showing total energy delivered over time with upper and lower bounds.]
Example 6: Island with Large Penetration of Renewables

- [James-Smith and Togeby 2007]
- Bornholm (DK) object of EcoGrid EU project
- Electricity: Peak demand 55 MW, Supply 30MW wind turbines, 60MW AC cable to mainland, one Combined Heat and Power plant (coal, 35 MW total)
- Issue: operation in islanded mode due to frequent cable cuts
  - Wind volatility
  - Generation may become large
  - Coal plant is not fast enough
  - ±3 MW of additional fast response (within 15 mn) is required

Example 6: Findings in [James-Smith and Togeby 2007]

- Demand response in homes (heating, hot water, refrigerators) can provide 3MW of capacity in winter
- Positive demand response (homes, district heating system) can avoid spilling wind energy
Example 7: Impact of e-car charging on distribution network [Clement-Nyns et al 2010]

- E-car charges are high power (4kW), stress electrical distribution network – peak demand at nights

### TABLE I
**RATIO OF POWER LOSSES TO TOTAL POWER [%] FOR THE 4 kW CHARGER IN CASE OF UNCOORDINATED CHARGING**

<table>
<thead>
<tr>
<th>Charging period</th>
<th>Penetration level</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>21h00-06h00</td>
<td>Summer</td>
<td>1.1</td>
<td>1.4</td>
<td>1.9</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>Winter</td>
<td>1.4</td>
<td>1.6</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>18h00-21h00</td>
<td>Summer</td>
<td>1.5</td>
<td>2.4</td>
<td>3.8</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Winter</td>
<td>2.4</td>
<td>3.4</td>
<td>4.8</td>
<td>5.0</td>
</tr>
<tr>
<td>10h00-16h00</td>
<td>Summer</td>
<td>1.3</td>
<td>1.8</td>
<td>2.6</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>Winter</td>
<td>1.7</td>
<td>2.2</td>
<td>3.0</td>
<td>3.6</td>
</tr>
</tbody>
</table>

### TABLE II
**MAXIMUM VOLTAGE DEVIATIONS [%] FOR THE 4 kW CHARGER IN CASE OF UNCOORDINATED CHARGING**

<table>
<thead>
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<th>Charging period</th>
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<tr>
<td></td>
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<td>6.3</td>
<td>8.5</td>
<td>10.3</td>
</tr>
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</tr>
<tr>
<td></td>
<td>Winter</td>
<td>3.7</td>
<td>4.9</td>
<td>6.4</td>
<td>7.7</td>
</tr>
</tbody>
</table>

Simulation of 34-bus residential grid [Clement-Nyns et al 2010]

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**Scheduled Charging**

- problem can be solved by *scheduling* the loads (e-cars), i.e. coordinate them
- e-cars communicate with a scheduler, through smart meter or other communication means
- coordinator solves optimization problem and sends schedule to e-car chargers

\[
\min \sum_{t=1}^{t_{max}} \sum_{n=1}^{n_{max}} R_t P_{t,n}^2 \\
\text{s.t.} \begin{cases}
\forall t, \forall n \in \text{nodes} : 0 \leq P_{n,t} \leq P_{\text{max}} \\
\forall n \in \text{nodes} : \sum_{t=1}^{t_{max}} P_{n,t} x_n = C_{\text{max}} \\
x_n \in [0, 1], \\
\text{power scheduled to car n at time t}
\end{cases}
\]

- requires: model of grid; of state and availability of e-cars; is frequently recomputed to address stochastic changes
Scheduled charging can eliminate need to upgrade distribution network

![Graph showing voltage profile](image)

Say what is true

Demand Response can be used...
1. ... to mitigate the impact of a weak grid
2. ... to compensate for energy imbalance
3. ... as an alternative to nuclear energy

![Bar chart showing demand response options](image)
**Say what is true**

1. Demand response can decrease the cost of electricity by reducing the required peak capacity
2. Voltalis makes money by selling Negawatts
3. Demand response may increase the cost of electricity in some time slots.

```
1. 1
2. 2
3. 3
4. 1 and 2
5. 1 and 3
6. 2 and 3
7. All
8. None
9. I don’t know
```

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### Taxonomy of Demand Response

**Type of user contract**

1. Time of use (e.g. day versus night)
2. Control by tariff (dynamic prices)
3. Control by quantity (interruptible supply, schedules)

**Mode of communication**

1. Inband tones (Ripples)
2. Powerline communication and smart meters
3. Radio communication

**Time scale of operation**

1. Static
2. Dynamic
   - 5mn-24 hours (smart meters)
3. Real time
   - (frequency response)

**Global Effect**

4. Shift the load (delay or anticipate)
5. Reduce demand (emergency, shave the peak on exceptional days)
QUESTIONS?

ELEMENTS OF THEORY

1. Demand and Supply Curves
2. Elasticity
3. Evaporation
4. Earliest and latest schedules
1. The Economic Theory of Demand Response

Consumer Side

- The economic theory of Demand Response is based on the following model.
- Assume consumers are willing to consume some amount of energy \( q \) at a price \( p \); in a given time slot, the utility of \( q \) is assumed to be measurable and equal to \( U(q) \); the consumer chooses the value of \( q \) that maximizes \( U(q) - pq \).

$$U(q)$$

non elastic load  
elastic load  
elastic load with minimum requir’t

The Economic Theory of Demand Response

Supplier Side

- Assume suppliers users are willing to sell some amount of energy \( q \) at a price \( p \); in a given time slot, the running cost of generating \( q \) is assumed to be measurable and equal to \( C(q) \); the supplier chooses the value of \( q \) that maximizes \( pq - C(q) \).

$$C(q)$$

wind supplier  
flexible supplier  
flexible supplier with maximum capacity
Demand and Supply Curves

- **Demand Curve** = how much consumer is willing to buy at a given price
- **Supply curve** = how much supplier is willing to sell at a given price

- Consumer maximizes $U(q) - pq$ therefore $U'(q) = p$
- Supplier maximizes $pq - C(q)$ therefore $C'(q) = p$

- Demand curve is $q \rightarrow U'(q)$
- Supply curve is $q \rightarrow C'(q)$

- $U$ concave $\Rightarrow U'$ is decreasing
- $C$ convex $\Rightarrow C'$ is increasing

Market Equilibrium

- Assume there is a perfect market to fix prices; the supplier and consumer prices are equal
- Price and quantity are equal by intersection of supply and demand curves
**Supply and Demand Curves Without Demand Response**  
[Kirschen 2003]

- No demand response means loads are inelastic; generation or grid outages cause prices to surge.

- Elastic loads may avoid price peaks.

---

**Assume some loads disconnect when price becomes \( p > p_0 \)**  
Which curve could be a demand curve for the aggregate demand?

1. Curve 1
2. Curve 2
3. Curve 3
4. Either 1 or 2
5. Either 1 or 3
6. Either 2 or 3
7. All
8. None
9. Ne znam
Norway’s pilot study [Saele and Grande 2011]
Demand Response may reduce prices

- 120 EUR/MWh difference between 2 areas inside Norway
- [Saele and Grande 2011] claims that the price peak would be suppressed with demand response

Supply Curve for Industrial Customers

Figure 3-k. Demand response of large industrial Hour-Ahead customers in Georgia Power’s RTP program. Scales are logarithmic. We have added on the x-axis a few price levels in $ per kWh.
Source: Braithwait, Christensen and Associates
2. Elasticity

Do we get this? ... or that?

Figure 3: It is a conceptual illustration of the response of a building to CPP on a hot afternoon. The example assumes CPP is invoked from 15:00 to 17:00. The figure shows two different usage patterns in a single sketch. Pattern 1 (Normal Load) is a typical effect, where loads drop at about 5 pm. For Pattern 2, the air conditioning demand actually increases after 5 pm, because the thermostat has been set back down to 72°F.

Source: Pat McLaughlin, CTC

Elasticity and Cross-Elasticity

- Demand response causes demand reduction and time shifting
- The quantitative effect is captured by

\[ (\text{self})\text{-elasticity} = \frac{dq}{dp} \frac{p}{q} = \frac{d(\log p)}{d(\log q)} \]

and

\[ (\text{cross-elasticity}) E_{t+h,t} := \frac{\partial q_{t+h}}{\partial p_t} \frac{p_t}{q_{t+h}} \]

defined for example for \( h \in [-24 \text{hours}, +24 \text{hours}] \)
Example of Cross-Elasticity

[Kirschen et al 2000]

- Users expect some prices $p_t$ based on historical data.
  Resulting demand is $q_t$.
  Assumes two demand response models with cross-elasticity.

- Market decides for different prices, $\Delta p_t = \text{difference between expected price and actual price}$. Demand response cause users to change their loads. [Kirschen et al 2000] assumes that

$$\Delta q_t = \sum_{h=-24}^{+24} \frac{\Delta p_{t+h}}{p_{t+h}} \varepsilon_{t,t+h} q_{t+h}$$

where $\varepsilon_{t,t+h}$ is called the Cross-Elasticity Coefficient

(it slightly differs from $E_{t,t+h}$)

$\varepsilon_{t,t+h} \times \frac{\Delta p_{t+h}}{p_{t+h}}$ is the fraction of the load at time $t+h$ that is moved to time $t$ due to a change in price at time $t+h$.

Example of Cross-Elasticity Coefficients

- $\Delta q_t = \sum_{h=-24}^{+24} \frac{\Delta p_{t+h}}{p_{t+h}} \varepsilon_{t,t+h} q_{t+h}$
- [Kirschen et al 2000] considers two possible scenarios:

  **Scenario 1:** (Time Shifting, “Inflexible”):
  
  $\varepsilon_{t-3,t} = \varepsilon_{t-2,t} = \varepsilon_{t-1,t} = +0.0033$
  
  $\varepsilon_{t+3,t} = \varepsilon_{t+2,t} = \varepsilon_{t+1,t} = +0.0033$
  
  $\varepsilon_{t,t} = -0.20$

  i.e. change in price at $t$ changes load by $-0.2 \times \%$ price increase load is transferred to 3 hours before and 3 hours after $t$

  **Scenario 2:** (“Optimizer”):
  
  $\varepsilon_{0,t} = \cdots = \varepsilon_{2,t} = \varepsilon_{16,t} = \cdots = \varepsilon_{23,t} = +0.01$
  
  $\varepsilon_{8,t} = \cdots = \varepsilon_{7,t} = +0.025$
  
  $\varepsilon_{t,t} = -0.20$

  i.e. change in price at $t$ changes load by $-0.2 \times \%$ price increase most load is transferred to early and late hours of the day.
Impact on Price

- Assuming no elasticity, prices are formed by matching demand, let \( \tilde{q} \mapsto \tilde{p} = \tilde{F}(\tilde{q}) \) the process of price formation, where \( \tilde{p} = (p_0, p_1, \ldots, p_{23}) \).

- [Kirschen et al 2000] studies a case with normal operation and with planned loss of generator.

Impact on Price (continued)

- Assume now elastic loads with known cross-elasticity. The actual load depends on the market price: let \( \tilde{p} \mapsto \tilde{q} = \tilde{G}(\tilde{p}) \) be the process of load adaptation.

- Assume market aggregator knows elasticity; she can compute market prices by solving a fixed point problem,

\[
\begin{cases}
\tilde{p} = \tilde{F}(\tilde{q}) \\
\tilde{q} = \tilde{G}(\tilde{p})
\end{cases}
\]

Fig. 7. Initial prices and prices as modified by elasticities. [Kirschen et al 2000]
3. Evaporation

- Evaporation = fraction of energy that is saved due to demand response [Le Boudec and Tomozei 2013]

\[
evaporation = \frac{E_0 - E_1}{E_0}
\]

What can we say about the evaporation for this scenario?

1. $> 0$
2. $< 0$
3. $= 0$
4. Nothing, it depends on other factors.
5. Não sei
**Evaporation**

- Evaporation = fraction of energy that is saved due to demand response [Le Boudec and Tomozei 2013]

\[
\text{evaporation} = \frac{E_0 - E_1}{E_0}
\]

- with pure demand shifting, evaporation = 0
- If it is true that demand response saves energy, we should see evaporation > 0
- What do we expect in general?

---

*(Should I keep my chalet warm?)*

**When I am away I interrupt heating. Does this save energy?**

1. Yes, there must be a reduction in total energy consumed
2. No, there cannot be any reduction in total energy consumed
3. Total energy consumed is increased
4. I weiss nid
We have seen this question already…

Voltalis does not pay nor charge anything to consumers but claims that consumers benefit by seeing a reduced electricity bill. Do you think this is true?

1. Yes, there must be a reduction in total energy consumed
2. No, there cannot be any reduction in total energy consumed
3. Total energy consumed is increased
4. I don’t know

(Should I keep my chalet warm?) When I am away I interrupt heating. Does this save energy?

1. Yes, there must be a reduction in total energy consumed
2. No, there cannot be any reduction in total energy consumed
3. Total energy consumed is increased
4. I don’t know

<table>
<thead>
<tr>
<th>Yes, there must be a reduction in total energy consumed</th>
<th>59%</th>
</tr>
</thead>
<tbody>
<tr>
<td>No, there cannot be any reduction in total energy consumed</td>
<td>34%</td>
</tr>
<tr>
<td>Total energy consumed is increased</td>
<td>28%</td>
</tr>
<tr>
<td>I don't know</td>
<td>0%</td>
</tr>
</tbody>
</table>
**Evaporation is not the same as “Rebound Effect”**

**Q1.** Does shutting down the heating today imply reducing total energy consumption compared to keeping temperature constant? = is evaporation positive?  
**A.** We will see later.

**Q2.** Does shutting down the heating today (and switching it off tomorrow) imply increasing tomorrow’s energy consumption?  
**A.** Yes (this is the rebound effect).

---

Assume the house model of [McKay 2008]

\[
d(t) = K_{leak} (T(t) - \theta(t)) + C_{outside} (T(t) - T(t-1))
\]
Heat provided to building:

\[ d(t) = K(T(t) - \theta(t)) + C(T(t) - T(t-1)) \]

Sum over \( t \) from 1 to \( \tau \):

Efficiency \( E \), total energy provided:

\[ \sum_{t=1}^{\tau} d(t) = K \sum_{t=1}^{\tau} (T(t) - \theta(t)) + C(T(\tau) - T(0)) \]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>No interruption</th>
<th>With interruption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building temperature</td>
<td>( T^*(t), t = 0 \ldots \tau )</td>
<td>( T(t), t = 0 \ldots \tau, ) ( T(t) \leq T^*(t) )</td>
</tr>
<tr>
<td>Heat provided</td>
<td>( E^* = \frac{1}{\varepsilon} \left( K \sum_{t=1}^{\tau} (T^<em>(t) - \theta(t)) + C(T^</em>(\tau) - T^*(0)) \right) )</td>
<td>( E &lt; E^* )</td>
</tr>
</tbody>
</table>
Q1. Does shutting down the heating today imply reducing total energy consumption compared to keeping temperature constant? = is evaporation positive? A. yes, it reduces energy consumption, due to less leakage

The French ADEME agency finds that consumers with Voltalis’s load switching devices save $\approx 10\%$ on heating but there is no significant saving on hot water boilers [ADEME 2012]. How do you interpret this?

1. The model we saw is too simple and its finding do not apply.
2. Boiler leakage is small, house leakage is not.
3. House leakage is small, boiler leakage is not.
4. Hot water boiling is negligible consumption compared to house heating
5. I don’t know.
Evaporation

- Resistive heating system with poorly insulated building: evaporation is positive.

- If heat = heat pump, coefficient of performance $\epsilon$ may be variable. Evaporation may be positive or negative; negative evaporation is possible (heat pump operating at night in cold air).

- Electric vehicle: we expect evaporation = 0 (pure time shifting). However charge intensity impacts losses; fast charging may consume more energy, negative evaporation is possible.

4. Earliest and Latest Schedules

- Assume market aggregator schedules energy consumption

- Assume evaporation = 0 (e.g. boilers, e-cars with not too high charge intensity)

- Then there are always earliest and latest schedules, and these can be computed, as we see next
A Simple Storage Problem

- Assume an infinite buffer into which we store some goods (e.g., energy); \( z(t) \) units of good are stored during slot; \( z(t) \) is known.
- We have to decide \( u(t) \), how many units of good we output at time \( t \). We have to satisfy the constraint \( u(t) \leq a(t) \), where \( a(t) \) is known.
- Let \( U(t) \) be the total amount output from time 0 to time \( t \).
- What is the \( U() \) that corresponds to the most aggressive output?

---

**Example**

\( a(t) = 1 \) unit of good / unit of time 

Storage is empty at time 0

\[ Z(t) = \text{cumulative input} = z(1) + \cdots + z(t) \]

\[ U(t) = u(1) + \cdots + u(t) \]

---

**Diagram**

- \( z(t) \) is instant input
- \( U(t) \) is cumulative input
- \( a(t) = 1 \) is a possible output?
Is \( u_1() \) a feasible output for my storage problem?

1. Yes
2. No
3. It depends on other elements not shown on picture
4. I don’t know

Solution

\( a(t) = 1 \) unit of good / unit of time

storage is empty at time 0

\( Z(t) = \) cumulative input

\( = z(1) + \cdots + z(t) \)

is \( u_1(t) \) a possible output?
Storage system are best studied using cumulative input and output curves

- $u()$ is a feasible output iff
  \[
  0 \leq u(t) \leq a(t) \\
  U(t) \leq Z(t)
  \]
  where $U(t) = u(1) + \cdots + u(t)$

- The Most Aggressive Output $U^*()$ is the one that minimizes storage content at any time, given the constraints on output rate $a(t)$
- satisfies $U_1(t) \leq U^*(t)$ for any other feasible output $U_1$
- can be computed online (i.e. is causal) by
  \[
  u^*(0) = 0, \\
  u^*(t) = \min\{a(t), \; Z(t) - U^*(t) - 1\}, \; t = 1, 2, \ldots
  \]
The Maximum Solution Theorem

Consider the problem
\[
\begin{align*}
0 & \leq u(t) \leq a(t), t = 1, 2, \\
U(t) & \leq Z(t), t = 0, 1, ...
\end{align*}
\]
with \( U(0) \leq Z(0) = 0 \) and \( u(t) := U(t) - U(t - 1) \). Here \( U() \) is the unknown function and the functions \( Z(), a() \) are known. i.e. we have constraints on the function \( U() \) and its discrete time derivative \( u() \)

- This problem has one unique maximum solution \( U^* \), i.e. \( U^* \) is a solution and for any other solution \( U \), we have \( U(t) \leq U^*(t), \forall t \)
- \( U^*(t) \) can be defined by causal iteration on time:
  \[
  u^*(0) = 0 \\
  u^*(t) = \min\{a(t) \text{, } Z(t) - U^*(t - 1)\}
  \]
- The proof is based on the formula \([\text{Le Boudec and Thiran 2001}]\)
  \[
  U^*(t) = \min_{s=0,...,t} \{Z(s) + a(s + 1) + \cdots + a(t)\}
  \]

Another Simple Storage Problem

- Assume an infinite buffer into which we store some goods (e.g. energy); \( v(t) \) units of good are stored during slot; \( v(t) \) is to be decided. The initial storage content \( V(0) \) is also to be decided.
- We have to satisfy the constraint \( v(t) \leq a(t) \), where \( a(t) \) is known.
- We have to output \( z(t) \) at any time slot \( t = 1, \ldots, T \), where \( z(t) \) is known.
- Let \( V(t) \) be the total amount input from time 0 to time \( t \).
- What is the \( V() \) that corresponds to the laziest input (i.e. as late as possible)?
The Laziest Input $V_s()$...

$$a(t) = 1 \ \text{unit of good} / \ \text{unit of time}$$

$$Z(t) = \text{cumulative output} = z(1) + \cdots + z(t)$$

$$z(t) = \text{instant output}$$

Say what is true...

1. $V_1()$ is the laziest input
2. $V_2()$ is the laziest input
3. None of them is the laziest input
4. I don’t know
Solution

$$a(t) = 1 \text{ unit of good / unit of time}$$

$$Z(t) = \text{cumulative output} = z(1) + \cdots + z(t)$$

- $V_1()$ is a feasible input but is not the laziest (can be delayed)
- $V_2()$ is not a feasible input
- The laziest input is drawn backwards in time

The Laziest Input $V^*(())$

- is the one that minimizes storage content at any time, given the constraints on output rate $a(t)$
- satisfies $V_1(t) \geq V^*(t)$ for any other feasible input $V_1$
- can be computed \textit{backwards in time}:
  $$V^*(T) = Z(T),$$
  $$v^*(t) = \min\{a(t), V^*(t) - Z(t - 1)\}, t = T, T - 1, \ldots, 1$$
  $$V^*(t - 1) = V^*(t) - v^*(t)$$
The Minimum Solution Theorem

Consider the problem:

\[
\begin{align*}
0 & \leq v(t) \leq a(t), t = 1, 2, ..., T, \\
V(t) & \geq Z(t), t = 0, 1, ..., T,
\end{align*}
\]

with \( V(T) \geq Z(T) \geq Z(0) = 0 \) and \( v(t) := V(t) - V(t - 1) \). Here \( V() \) is the unknown function and the functions \( Z(), a() \) are known.

This problem has one unique minimum solution \( V^*_t \), i.e. \( V^*_t \) is a solution and for any other solution \( V \), we have \( V(t) \geq V^*_t(t), \forall t = 0, ... T \).

\( V^*_t() \) can be defined by backwards iteration on time:

\[
\begin{align*}
V^*_t(T) & = Z(T), \\
v^*_t(t) & = \min(a(t), V^*_t(t) - Z(t - 1)), t = T, T - 1, ... 1, \\
V^*_t(t - 1) & = V^*_t(t) - v^*_t(t).
\end{align*}
\]

The proof is based on the formula [Le Boudec and Thiran 2001]

\[
V^*_t(t) = \max_{s=1,...,t} \left( Z(s) - a(t + 1) - \cdots - a(s) \right)
\]

The Energy Scheduling Problem

Assume you want to schedule energy delivery to a storage (e.g. boiler) over a period \([0, T]\).

The problem is to schedule \( u(t) \), energy in slot \( t \).

The anticipated consumption \( j(t) \) (hot water) is assumed to be known.

The constraints on the system are:

1. \( 0 \leq u(t) \leq a(t) \) power limit
2. \( J(t) \leq U(t) + B_0 \) consumption constraint
3. \( U(t) - J(t) + B_0 \leq B_{\text{max}} \) no overflow

\( B_0 \) = storage level at \( t = 0 \)
\( B_{\text{max}} \) = storage capacity

Example 5: Boilers as Tertiary Reserve [Sundstrom et al 2012]
The earliest and latest schedules

A feasible schedule is constrained by (1) and (3) maximum solution theorem
\[ U(t) \leq U^*(t) \]
where \( U^*(t) \) is the most aggressive, i.e. earliest schedule (trying to keep the storage full)

\[ U^*(t) \] can be computed iteratively:
\[ u^*(t) = \min\{a(t), B_{\text{max}} - B_0 + J(t-1) - U^*(t)\} \]

A feasible schedule is constrained by (1) and (2) minimum solution theorem
\[ U(t) \geq U_*(t) \]
where \( U_*(t) \) is the laziest (i.e. latest) schedule (trying to deliver energy as late as possible)

\[ U_*(t) \] can be computed iteratively backwards in time, starting from \( U_*(T) = J(T) - B_0 \):
\[ u_*(t) = \min\{a(t), -B_0 + J(t-1) - U_*(t)\} \]
\[ U_*(t-1) = U_*(t) - u_*(t) \]
The earliest and latest schedules

**Theorem:** A tentative schedule \( u(t) \) is feasible if and only if it satisfies (1) and
\[
U_\alpha(t) \leq U(t) \leq U^*(t)
\]

(1) \( 0 \leq u(t) \leq a(t) \)  
- power limit

(2) \( f(t) \leq U(t) + B_0 \)  
- consumption constraint

(3) \( U(t) - f(t) + B_0 \leq B_{\text{max}} \)  
- no overflow

\( B_0 = \) storage level at \( t = 0 \)
\( B_{\text{max}} = \) storage capacity

\[ \begin{align*}
\text{grid cells for reserve} & \quad \text{upper bound} \quad \text{total energy available} \\
\text{500 miles of travel} & \quad \text{peak heating for 4 hours} \\
\text{20 min} & \quad \text{4 hr}
\end{align*} \]

---

**Conclusion**

- Demand Response adapts loads to cope with variability
- Is required as long as storage of electricity is expensive
- Can use pricing or control by quantity

- Network problem involves economic theory and scheduling

- User problem involves model predictive control (MPC)  
  -- see next lecture
References


References (continued)

- [Kirschen et al 2000] Daniel S. Kirschen, Goran Strbac, Pariya Cumperayot, and Dilemar de Paiva Mendes “Factoring the Elasticity of Demand in Electricity Prices”
- [Sundstrom et al 2012] Sundstrom, O.; Binding, C.; Gantenbein, D.; Berner, D.; Rumsch, W.-C. “Aggregating the flexibility provided by domestic hot-water boilers to offer tertiary regulation power in Switzerland”, ISGT Europe 2012
**ISSUES WITH DEMAND RESPONSE**

[Le Boudec and Tomozei 2013]

**Issue with Demand Response: Grid Changes Load**

- Widespread demand response may make load hard to predict

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**Intention**

- renewables

**Real**

- load with demand response
  - natural load
A Macroscopic Model of Demand Response

Macroscopic Model of [Cho and Meyn 2010]

\[ D^n(t) = D^1(t) + D(t) \]

\[ G^n(t) = G(t - 1) + G(t) + M(t) \]

Control

Supply

Natural Demand

Expressed Demand

Satisfied Demand

Reserve (Excess supply)

\[ F(t) = [E^n(t) - G^n(t)]^+ \]

\[ R(t) = G^n(t) - E^n(t) \]

Evaporation

Randomness

Ramping Constraint

\[ -\xi \leq G(t) - G(t - 1) \leq \zeta \]

The Control Problem

- **Control variable:**
  \[ G(t - 1) \]
  production bought one time slot ago in real time market

- **Controller sees only supply**
  \[ G^a(t) \]
  and expressed demand \[ E^a(t) \]

- **Our Problem:**
  keep backlog \( Z(t) \) stable

- **Ramp-up and ramp-down constraints**
  \[ \xi \leq G(t) - G(t - 1) \leq \zeta \]
Threshold Based Policies

\[ G^f(t) = D^f(t) + r_0 \]  
Forecast supply is adjusted to forecast demand

\[ R(t) = G^a(t) - E^a(t) \]  
R(t) := reserve = excess of demand over supply

Threshold policy:
- if \( R(t) < r^* \) increase supply to come as close to \( r^* \) as possible (considering ramp up constraint)
- else decrease supply to come as close to \( r^* \) as possible (considering ramp down constraint)

Simulations (evaporation \( \mu > 0 \))
Simulations (evaporation $\mu > 0$) $r^*$

- $\mu > 0$ means returning load is, in average, less
- Large excursions into negative reserve and large backlogs are typical and occur at random times

Large backlogs may occur within a day, at any time (when evaporation $\mu > 0$)

Typical delay $\frac{1}{A} = 30$ mn, all simulations with same parameters as previous slide, $\sigma = 160$
Findings: Stability Results

- If evaporation $\mu$ is positive, system is stable (ergodic, positive recurrent Markov chain) for any threshold $r^*$

- If evaporation $\mu$ is negative, system unstable for any threshold $r^*$

- Delay does not play a role in stability
- Nor do ramp-up / ramp down constraints or size of reserve

What this suggests about Demand Response:

- Positive evaporation is essential occurs with thermal loads, might not always occur for all load
- Model suggests that large backlogs are possible and unpredictable

- Backlogged load is a new threat to grid operation
  Need to measure and forecast backlogged load