Real-Time Control of Electrical Distribution Grids

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\textsuperscript{1} https://people.epfl.ch/105633/research
\textsuperscript{2} http://smartgrid.epfl.ch
Credits

Joint work
EPFL-DESL (Electrical Engineering)
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Contents

1. Real-time operation of electrical distribution grids (COMMELEC)

2. V-control
1. Real-Time Operation of Microgrid: Motivation

Absence of inertia (inverters)
Stochastic generation (PV)
Storage, demand response
Grid stress (charging stations, heat pumps)
Support main grid (primary and secondary frequency support)

⇒ Agent based, real-time control of microgrid
COMMELEC Uses Explicit Power Setpoints

Every 100 msec
- Grid Agent monitors grid and sends power setpoints to Resource Agents
- Resource agent sends to grid agent: PQ profile, Virtual Cost and Belief Function

Goal: manage quality of service in grid; support main grid; use resources optimally.

[Bernstein et al 2015, Reyes et al 2015]
https://github.com/LCA2-EPFL/commelec-api

PQ profile = set of setpoints that this resource is willing to receive
Belief Function

Say grid agent requests setpoint \((P_{\text{set}}, Q_{\text{set}})\) from a resource; actual setpoint \((P, Q)\) will, in general, differ.

**Belief function** exported by resource agent means: the resource implements \((P, Q) \in BF(P_{\text{set}}, Q_{\text{set}})\)

Quantifies uncertainty due to nature + local inverter controller

Essential for safe operation
Operation of Grid Agent

Grid agent computes a setpoint vector $x$ that minimizes

$$J(x) = \sum_i w_i C_i(x_i) + W(z) + J_0(x_0)$$

subject to admissibility.

$x$ is admissible $\Leftrightarrow (\forall x' \in BF(x), \ x' \text{ satisfies security constraints})$
Implementation / EPFL Microgrid

Topology: 1:1 scale of the Cigré low-voltage microgrid benchmark TF C6.04.02 [Reyes et al, 2018]

- Phasor Measurement Units: nodal voltage/current syncrophasors @50 fps
- Solar PVs on roof and facade
- Battery
- Thermal Load (flex house)
Dispatch and Primary-Frequency Support

Superposition of dispatch and primary frequency control (i.e., primary droop control) with a max regulating energy of 200 kW/Hz

In parallel, keep the internal state of the local grid in a feasible operating condition.
COMMELEC Uses Active Replication with Real-Time Consensus

iPRP: transparent duplication of IP multicast and redundant networks
Axo: makes sure delayed messages are not used
Quarts: grid agents perform agreement on input
Added latency ≤ one RTT – compare to consensus’s unbounded delay

[Mohiuddin et al 2017, Saab et al 2017]
https://github.com/LCA2-EPFL/iPRP
2. Controlling the Electrical State with Uncertain Power Setpoints

**Admissibility test**: when issuing power setpoint $s$, grid agent tests whether the grid is safe during the next control interval for all power injections in the set $S = BF(s)$. 
Load Flow Mapping

Electrical state $\nu \in \mathbb{C}^{3N}$: collection of complex phasors
Power injection $s \in \mathbb{C}^{3N}$: collection of complex powers injected (generated or consumed) at all nodes

Load flow mapping $s = F(\nu)$ is quadratic.
Inverse problem “find $\nu$ given $s$” has 0 or many solutions.

Security constraints are constraints on $\nu$ bearing on voltage and currents + non-singularity of $\nabla F_\nu$
Controlling the Electrical State with Uncertain Power Setpoints

Admissibility test: when issuing power setpoint $s$, grid agent tests whether the grid is safe during the next control interval for all power injections in the set $S = BF(s)$.

The abstract problem is:

- given an initial electrical state $v$ of the grid
- given that the power injections $s$ remain in some uncertainty set $S$ can we be sure that the resulting state of grid satisfies security constraints and is non-singular?
$\mathcal{V}$-Control

$S$ is a domain of $\mathcal{V}$-control $\iff$ whenever $t \mapsto \nu(t)$ is continuous, knowing that $\nu(0) \in \mathcal{V}$ and $\forall t \geq 0, F(\nu(t)) \in S$ ensures that $\forall t \geq 0, \nu(t) \in \mathcal{V}$.

[Wang et al 2017b]

3-phase grid with one slack bus and $N$ PQ buses; $\nu = \text{electrical state} = \text{complex voltage at all non slack buses}$; $s = \text{power injection vector at all non slack buses}$
Existence of Load Flow Solution Does not Imply V-control

For $S$ to be a domain of $V$-control it is necessary that every $s \in S$ has a load-flow solution in $V$.

But this is not sufficient.
Every $s \in \mathcal{S}$ has a load-flow solution in $\mathcal{V}$.
But starting from $s^0$ and $\nu = \diamond$ we exit $\mathcal{V}$.

$\mathcal{V} = \{\nu: |v_1|, |v_2| \in [0.9; 1.1] \text{ and } \nabla F_\nu \text{ non singular}\}$
$\mathcal{S} = \{s = \kappa(s_1^0, s_2^0), \kappa \in [0.992; 1]\}$
$\nu = \diamond$ is in interior of $\mathcal{V}$, close to boundary (in $s_1$)
Unique Load Flow Solution Does not Imply V-control

Assume that every $s \in S$ has a unique load-flow solution in $\mathcal{V}$.

This is **not sufficient** to guarantee that $S$ is a domain of $\mathcal{V}$-control.
Every $s \in \mathcal{S}$ has a unique load-flow solution in $\mathcal{V}$. But starting from $s^0$ and $\nu = \odot$ we exit $\mathcal{V}$.

$\mathcal{V} = \mathcal{V}^A \cup \mathcal{V}^B$

$\mathcal{S} = \{ s = \kappa(s_1^0, s_2^0), \kappa \in [0.999; 1] \} = \mathcal{S}^A \cup \mathcal{S}^B$

$\mathcal{S}^A = \{ s = \kappa(s_1^0, s_2^0), \kappa \in (0.999915; 1] \}$

$\mathcal{S}^B = \{ s = \kappa(s_1^0, s_2^0), \kappa \in [0.999; 0.999915] \}$
Sufficient Condition for V-control

**Theorem 3** in [Wang et al 2017b]

If

1. \( \mathcal{V} \) is open in \( \mathbb{C}^{3N} \)
2. \( \mathcal{S} \) is open in \( \mathbb{C}^{3N} \)
3. \( \forall s \in \mathcal{S} \) there is a unique load-flow solution in \( \mathcal{V} \) then \( \mathcal{S} \) is a domain of \( \mathcal{V} \)-control.

In the previous example, neither \( \mathcal{V} \) nor \( \mathcal{S} \) is open.
V-control and Non-Singularity

We call $\nu$ non-singular if $\nabla F_\nu$ is non-singular.

**Theorem 3** in [Wang et al 2017b]

If $\mathcal{V}$ is open in $\mathbb{C}^{3N}$

1. $\mathcal{V}$ is open in $\mathbb{C}^{3N}$
2. $\mathcal{S}$ is open in $\mathbb{C}^{3N}$
3. $\forall s \in \mathcal{S}$ there is a unique load-flow solution in $\mathcal{V}$

then $\mathcal{S}$ is a domain of $\mathcal{V}$-control.

Furthermore, every $\nu \in \mathcal{V}$ such that $F(\nu) \in \mathcal{S}$ is non-singular.
Uniqueness and Non-Singularity

We call \( \mathcal{V} \) a **domain of uniqueness** iff

\[ \forall \mathbf{v} \in \mathcal{V}, \forall \mathbf{v}' \in \mathcal{V}, \mathbf{v} \neq \mathbf{v}' \Rightarrow F(\mathbf{v}) \neq F(\mathbf{v}') \]

**Theorem 1** in [Wang et al 2017b]

If \( \mathcal{V} \) is open in \( \mathbb{C}^{3N} \) and is a domain of uniqueness then every \( \mathbf{v} \in \mathcal{V} \) is non-singular.

In this previous example, \( \mathcal{V} \) is not a domain of uniqueness
Grid Agent’s Admissibility Test

Problem (P): Given a set of power injections $S^{uncertain}$, find a set of electrical states $\mathcal{V}$ such that

1. $v(0) \in \mathcal{V}$
2. $\mathcal{V}$ is open
3. $\mathcal{V}$ is a domain of uniqueness
4. $\mathcal{V}$ satisfies security constraints (voltages and line currents)
5. $S^{uncertain} \subseteq F(\mathcal{V})$

By Theorems 1 and 3 (applied to $\mathcal{V}$ and $S = F(\mathcal{V})$), this will imply that $\mathcal{V}$ is non singular and $S^{uncertain}$ is a domain of $\mathcal{V}$-control.
Solving (P)

- Sufficient conditions for uniqueness and existence of load flow:
  **Theorem 1** in [Wang et al 2017a]
  Given is a load-flow pair \((\hat{v}, \hat{s})\). If \(\xi(s - \hat{s}) < \rho^\dagger(\hat{v})^2\) then \(s\) has a unique load flow solution in a disk around \(\hat{v}\) with radius \(\rho^\dagger(\hat{v})\). The norm \(\xi()\) and \(\rho^\dagger\) are derived from the Y matrix.

- Additional conditions (Def 3. in [Wang et al 2017b]) ensure security conditions.

- Domains can be patched (Thm 6 in [Wang et al 2017b])
**Notation [Wang et al 2017b]**

\[
\delta_j(\hat{v}, s) \triangleq \sum_{\ell=1}^{N} \left| \mathbf{\Gamma}_{j,\ell} \right| \| \text{diag}(\mathbf{w}_{\ell})^{-1} \| \eta_{\ell}(\hat{v}, s) \\
\frac{u_{\min}(\hat{v}) (u_{\min}(\hat{v}) - \rho^\dagger(\hat{v}, s))}{u_{\min}(\hat{v}) (u_{\min}(\hat{v}) - \rho^\dagger(\hat{v}, s))} ;
\]

(6)

zero-load nodal voltage \( \mathbf{w} \triangleq -\mathbf{Y}_{LL}^{-1}\mathbf{Y}_{L0}\mathbf{v}_0 \)

- \( \mathbf{\Gamma}_{j,\ell} \), \( j, \ell \in \mathcal{N}^{PQ} \) is the 3 \times 3 submatrix formed by rows \( \{3j-2, 3j-1, 3j\} \) and columns \( \{3\ell-2, 3\ell-1, 3\ell\} \) of \( \mathbf{Y}_{LL}^{-1} \);

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{W} )</td>
<td>diag(( \mathbf{w} ))</td>
</tr>
<tr>
<td>( \xi(s) )</td>
<td>( | \mathbf{W}^{-1}\mathbf{Y}<em>{LL}^{-1}\mathbf{W}^{-1}\text{diag}(\mathbf{s})|</em>{\infty} )</td>
</tr>
<tr>
<td>( u_{\min}(\mathbf{v}) )</td>
<td>( \min_{j\in\mathcal{N}^{PQ}, \gamma\in{a,b,c}}</td>
</tr>
<tr>
<td>( \rho^\dagger(\mathbf{v}) )</td>
<td>( \frac{1}{2} (u_{\min}(\mathbf{v}) - \xi(\mathbf{F}(\mathbf{v}))/u_{\min}(\mathbf{v})) )</td>
</tr>
<tr>
<td>( \rho^\dagger(\mathbf{v}, \mathbf{s}') )</td>
<td>( \rho^\dagger(\mathbf{v}) - \sqrt{\rho^\dagger(\mathbf{v})^2 - \xi(\mathbf{s}' - \mathbf{F}(\mathbf{v}))} )</td>
</tr>
<tr>
<td>( \eta_{\ell}(\mathbf{v}, \mathbf{s}') )</td>
<td>( u_{\min}(\mathbf{v})</td>
</tr>
</tbody>
</table>
Patching Example

The algorithm tries if a single \((\hat{v}, S)\) works, else breaks the set \(S\) into pieces and patches them.

IEEE 13-bus feeder, 3-phase configuration 602.

### Uncertainty set

<table>
<thead>
<tr>
<th>Round #</th>
<th>Bus 634 Phase b</th>
<th>Bus 671 Phase c</th>
<th>Bus 652 Phase a</th>
<th>Strictly secured pair</th>
<th>Consistent elements in (L)</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+ F((v_{initial}))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>No</td>
<td></td>
<td>Partition bus 671 phase c ; Add 4 elements to (L_{aux})</td>
</tr>
<tr>
<td>2</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>Yes</td>
<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
<tr>
<td>3</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>Yes</td>
<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
<tr>
<td>4</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>Yes</td>
<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
<tr>
<td>5</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>No</td>
<td></td>
<td>Partition bus 634 phase b ; Add 4 elements to (L_{aux})</td>
</tr>
<tr>
<td>6</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
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<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
<tr>
<td>7</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
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<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
<tr>
<td>8</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
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<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
<tr>
<td>9</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>+ F((v))</td>
<td>Yes</td>
<td>Yes</td>
<td>Add 1 element to (L)</td>
</tr>
</tbody>
</table>
Performance Evaluation

IEEE 37 bus feeder. $S_{uncertain} = [0, \kappa] \times$ benchmark values on all loaded phases. For $0 \leq \kappa \leq 1.15$ algorithm declares $S_{uncertain}$ safe in one partition and <20 msec runtime on one i7; for $\kappa > 1.15$ the algorithm needs multiple partitions but lowest voltage bound is close to limit.

IEEE 123 bus feeder. $S_{uncertain} = \left[1 - \frac{\kappa}{2}, 1 + \frac{\kappa}{2}\right] \times$ benchmark values on all loaded phases. For $0 \leq \kappa \leq .31$ algorithm declares $S_{uncertain}$ safe in one partition and <30 msec runtime; for $\kappa > .31$ the algorithm needs multiple partitions but highest branch current is close to limit.
Performance Evaluation

IEEE 37 bus feeder. One source added to one unloaded phase. Uncertainty set as shown. We limit the number of partitions to 8.

For $\kappa \leq 0.750$ no partition.

For $\kappa = 0.992$, 8 partitions and run-time $< 200$ msec. Low voltage bound is close.

Incidentally, lowest voltage is not at $(0,0)$ nor $(P_{\text{max}}, Q_{\text{max}})$ (non-monotonicity)
Conclusions

Controlling state of a grid by controlling power injections helps solve the problems posed by stochastic loads and generations.

Concrete implementations exist (COMMELEC) and use commodity hardware with solutions for active replication.

Accounting for uncertainty is essential. Testing admissibility of uncertain power setpoints can use the theory of V-control.
References

- http://smartgrid.epfl.ch