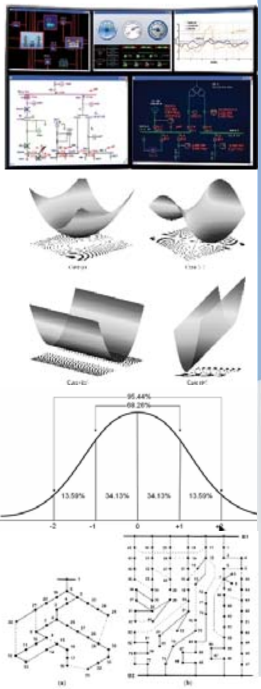

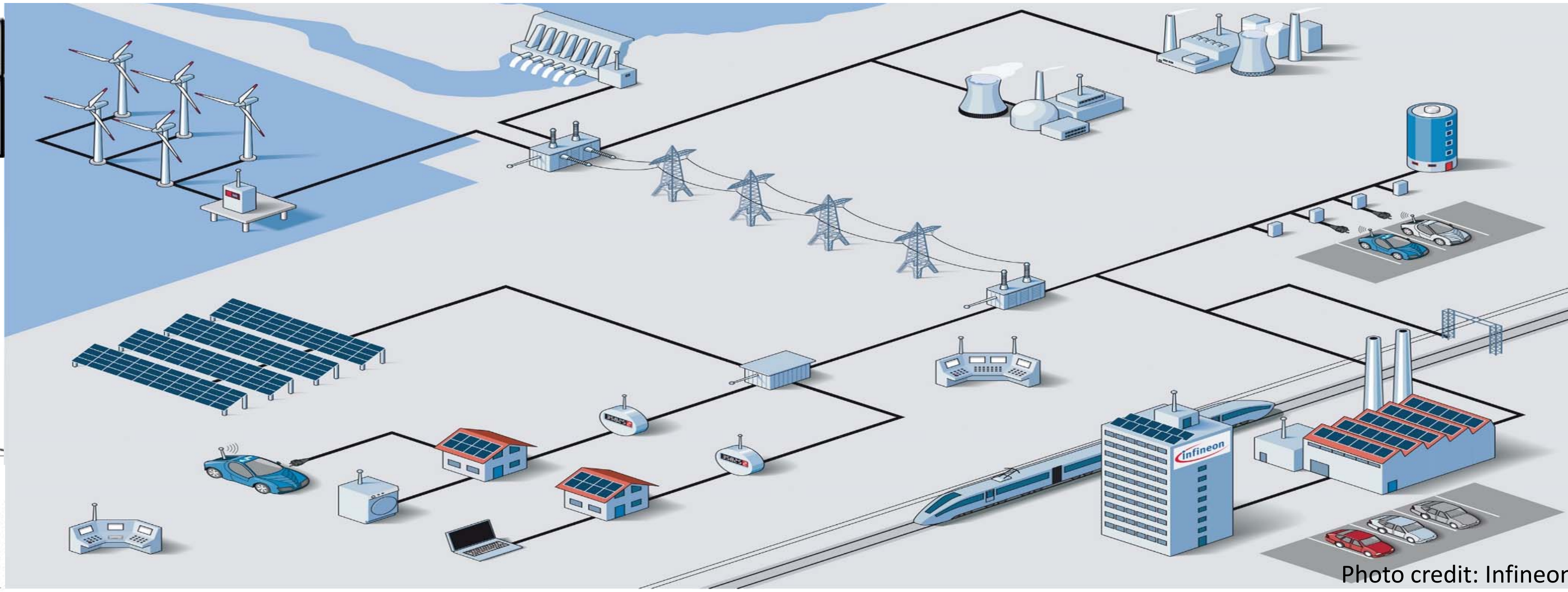

Photo credit: Infineon

The AR-OPF: an Exact Convex Formulation for the Optimal Power Flow in Radial Distribution Networks

Jean-Yves Le Boudec and Mario Paolone

EPFL LCA2 and DESL

(joint work with Dr. Mostafa Nick)

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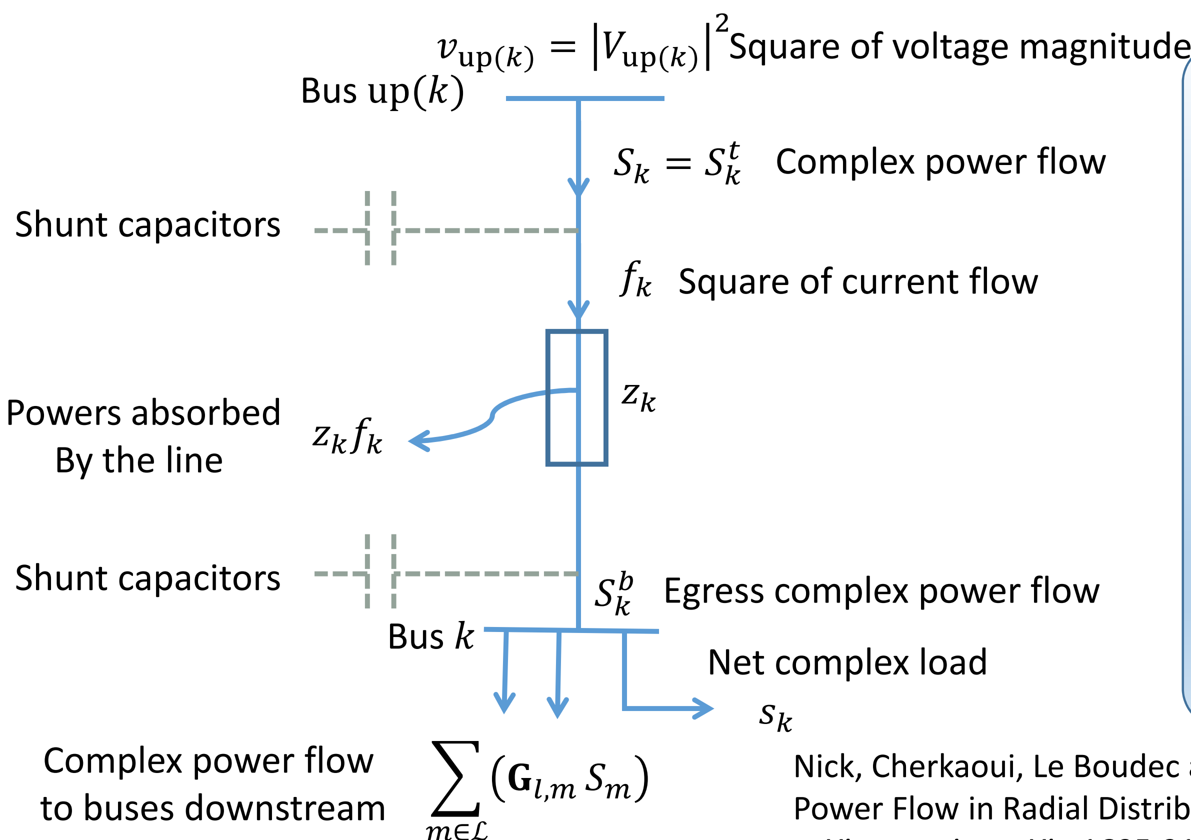
Power flow equations for radial distribution networks

With transverse parameters [Nick, Cherkaoui, Le Boudec and Paolone 2017]



Power flow equations

$$S_k = s_k + \sum_{m\in\mathcal{L}} (\mathbf{G}_{k,m} S_m) - \frac{j(v_{\mathrm{up}(k)} + v_k) b_l}{2} + z_k f_k$$

$$v_k = v_{\mathrm{up}(k)} - 2\Re\left(z_k^* S_k + \frac{j(v_{\mathrm{up}(k)}) b_k}{2}\right) + |z_k|^2 f_k$$

$$f_k = \frac{\left|S_k + \frac{j(v_{\mathrm{up}(k)}) b_k}{2}\right|^2}{v_{\mathrm{up}(k)}}$$

$\mathbf{G}_{k,m} = 1$ if $k = \mathrm{up}(m)$ else $= 0$

Nick, Cherkaoui, Le Boudec and Paolone 2017 "An Exact Convex Formulation of Optimal Power Flow in Radial Distribution Networks Including Transverse Components." *arXiv preprint arXiv:1605.01964* (2017).

Background of OPF in radial grids

Convex models

DistFlow

- **Linear** OPF model in radial distribution networks
  Neglects resistive **losses**
- **Transverse** parameters are **not considered**.

$$r_l f_l \approx 0$$

Branch flow relaxation

- **Transverse** parameters are **not** considered
- May **results** into physically **infeasible** solutions.

$$f_l \geq \frac{|S_l|^2}{v_{\mathrm{up}(l)}}$$

Linearization techniques

- **Linearization** of the LF
- **Coupling of the control decision with the operating point for the Jacobian computation.**

$$\Delta\left|\overline{V}_l(t)\right| = \mathrm{A}_l^P(t)\Delta P(t) + \mathrm{A}_l^Q(t)\Delta Q(t)$$

Original OPF (O-OPF) and its SOCP Relaxation (R-OPF)

O-OPF

$$\underset{s,S,v,l}{\text{minimize:}} \sum_{k \in \mathcal{L}} \Big(\mathcal{C}\big(\Re(s_k), \Im(s_k)\big) \Big) + \mathcal{C}^e(P_1^t)$$

Subject to:

1) Load flow equations

2) $v^{\min} \le v_k \le v^{\max}, \ \forall\, k \in \mathcal{L}$

3) $\frac{|s_k^t|^2}{v_{\mathrm{up}(k)}} \le I_k^{\max}, \ \forall\, k \in \mathcal{L}$

4) $\frac{|s_k^b|^2}{v_k} \le I_k^{\max}, \ \forall\, k \in \mathcal{L}$

5) $p_k^{\min} \le \Re(s_k) \le p_k^{\max}, \ \forall\, k \in \mathcal{L}$

6) $q_k^{\min} \le \Im(s_k) \le q_k^{\max}, \ \forall\, k \in \mathcal{L}$

We relax the non-convex constraint as in [Farivar, Clarke, Low and Chandy 2011].

$$f_k = \frac{\left| S_k + j\frac{v_{\mathrm{up}(k)} b_k}{2} \right|^2}{v_{\mathrm{up}(k)}} \Rightarrow f_k \ge \frac{\left| S_k + j\frac{v_{\mathrm{up}(k)} b_k}{2} \right|^2}{v_{\mathrm{up}(k)}} \Rightarrow$$

Relaxed Convex Optimal Power Flow (R-OPF)

$\mathcal{C}^e(P_1^t)$ is the cost function related to energy import from the grid and is strictly increasing. $\mathcal{C}\big(\Re(s_k), \Im(s_k)\big)$ is the cost function related to the nodal injection/consumption. Both $\mathcal{C}(.)$ and $\mathcal{C}^e(.)$ are assumed to be convex.

[Farivar, Clarke, Low and Chandy 2011]. "Inverter var control for distribution systems with renewables." *Smart Grid Communications (SmartGridComm), 2011*

Challenges with Relaxed Optimal Power Flow (R-OPF)

Challenge

Relaxation is inexact when any of these constraints bind:

- Nodal voltage-magnitude upper bound [Gan, Li, Topcu and Low 2015]
- Lines' ampacity-limit when accounting for shunt elements

Solution

- Define upper bound variables for nodal voltage magnitudes [Baran and Wu 1989] and for line currents / power flows
- Use these new variables in the line ampacity and nodal voltage-magnitude limits.

We define new auxiliary variables such that

$$\hat{P} \leq P \leq \bar{P}$$
$$\hat{Q} \leq Q \leq \bar{Q}$$
$$v \leq \hat{v}$$

Gan, Li, Topcu and Low 2015. "Exact convex relaxation of optimal power flow in radial networks." *IEEE TAC* 60.1 (2015): 72-87.
Baran and Wu 1989. "Optimal capacitor placement on radial distribution systems." *IEEE TPD* 4.1 (1989): 725-734.

Power flow equations in matrix form

$$\mathbf{D} = \mathbf{C}\big[2\text{diag}(r)\big((\mathbf{H}-\mathbf{I})\text{diag}(r)\big) + 2\text{diag}(x)\big((\mathbf{H}-\mathbf{I})\text{diag}(x)\big) + \text{diag}(|z|^2)\big]$$

$$\mathbf{F} = \left(\mathbf{H}\text{diag}(x) + \frac{1}{2}\mathbf{H}\text{diag}(B)\mathbf{D}\right)$$

$$\mathbf{E} = 2\text{diag}(\pi)\mathbf{H}\text{diag}(r) + 2\text{diag}(\varrho)\mathbf{F} + \text{diag}(\vartheta)\mathbf{D}$$

$$\hat{P} = \mathbf{H}p$$

$$\hat{Q} = \mathbf{H}q - \frac{1}{2}\mathbf{H}\text{diag}(b)\big(\mathbf{I} + \mathbf{G}^{\mathbf{T}}\big)\hat{v}$$

$$P = \hat{P} + \mathbf{H}\text{diag}(r)f$$

$$Q = \hat{Q} + \mathbf{H}\text{diag}(x)f + \frac{1}{2}\mathbf{H}\text{diag}(b)\big(\mathbf{I} + \mathbf{G}^{\mathbf{T}}\big)\mathbf{D}f$$

$z = r + \text{j}x$: Vector of lines' impedance

B: vector of susceptances connected to buses

$\mathbf{G}$: is the adjacency matrix of the oriented graph of the network

$\mathbf{H}$: is the closure of $\mathbf{G}$

$\mathbf{I}$: is the $L \times L$ identity matrix

$\mathbf{M} = \mathbf{2}\text{diag}(x)\ \mathbf{H}\ \text{diag}(B)$.

$\mathbf{C} = (\mathbf{I} - \mathbf{G}^T - \mathbf{M})^{-\mathbf{1}}$

$s = p, +jq$: vector of the power absorption at buses

f: the square of the current in the central part of line

Nick, Cherkaoui, Le Boudec and Paolone 2017 "An Exact Convex Formulation of Optimal Power Flow in Radial Distribution Networks Including Transverse Components." *arXiv preprint arXiv:1605.01964* (2017).

Proposed Convex OPF: Augmented Relaxed Optimal Power Flow (AR-OPF)

$$\underset{s,S,v,f,\hat{S},\hat{v},\bar{S},\bar{f}}{\text{minimize}} \sum_{l\in\mathcal{L}} \Big(\mathcal{C}\big(\Re(s_l),\Im(s_l)\big)\Big) + \mathcal{C}^e(P_1^t)$$

s.t.

$$S_l^t = s_l + \sum_{m\in\mathcal{L}} \mathbf{G}_{l,m} S_m^t + z_l f_l - j\big(v_{\mathrm{up}(l)} + v_l\big)b_l, \qquad \forall\, l\in\mathcal{L}$$

$$v_l = v_{\mathrm{up}(l)} - 2\Re\Big(z_l^*\big(S_l^t + jv_{\mathrm{up}(l)}b_l\big)\Big) + |z_l|^2 f_l, \forall l\in\mathcal{L}$$

$$S_l^b = s_l + \sum_{m\in\mathcal{L}} \mathbf{G}_{l,m} S_m^t, \qquad \forall\, l\in\mathcal{L}$$

$$p_l^{\min} \le \Re(s_l) \le p_l^{\max}, \qquad \forall\, l\in\mathcal{L}$$

$$q_l^{\min} \le \Im(s_l) \le q_l^{\max}, \qquad \forall\, l\in\mathcal{L}$$

$$f_l \ge \frac{\left|S_l^t + jv_{\mathrm{up}(l)}b_l\right|^2}{v_{\mathrm{up}(l)}}, \qquad \forall l\in\mathcal{L}$$

(*)

$$\hat{S}_l^t = s_l + \sum_{m\in\mathcal{L}} \mathbf{G}_{l,m}\hat{S}_m^t - j\big(\hat{v}_{\mathrm{up}(l)} + \hat{v}_l\big)b_l, \qquad \forall\, l\in\mathcal{L}$$

$$\hat{v}_l = \hat{v}_{\mathrm{up}(l)} - 2\Re\Big(z_l^*\big(\hat{S}_l^t + j\hat{v}_{\mathrm{up}(l)}b_l\big)\Big), \qquad \forall\, l\in\mathcal{L}$$

$$\bar{S}_l^t = s_l + \sum_{m\in\mathcal{L}} \mathbf{G}_{l,m} \bar{S}_m^t + z_l\bar{f}_l - j\big(v_{\mathrm{up}(l)} + v_l\big)b_l, \qquad \forall\, l\in\mathcal{L}$$

$$\bar{f}_l v_l \ge \max\left\{\left|\hat{P}_l^b\right|^2, \left|\bar{P}_l^b\right|^2\right\} + \max\left\{\left|\hat{Q}_l^b - \hat{v}_l b_l\right|^2, \left|\bar{Q}_l^b - v_l b_l\right|^2\right\}, \qquad \forall l\in\mathcal{L}$$

$$\bar{f}_l v_{\mathrm{up}(l)} \ge \max\left\{\left|\hat{P}_l^t\right|^2, \left|\bar{P}_l^t\right|^2\right\} + \max\left\{\left|\hat{Q}_l^t + \hat{v}_{\mathrm{up}(l)}b_l\right|^2, \left|\bar{Q}_l^t + v_{\mathrm{up}(l)}b_l\right|^2\right\}, \qquad \forall l\in\mathcal{L}$$

$$\bar{S}_l^b = s_l + \sum_{m\in\mathcal{L}} \mathbf{G}_{l,m} \bar{S}_m^t, \qquad \forall\, l\in\mathcal{L} \qquad \hat{S}_l^b = s_l + \sum_{m\in\mathcal{L}} \mathbf{G}_{l,m}\hat{S}_m^t, \qquad \forall\, l\in\mathcal{L}$$

$$v^{\min} \le v_l, \qquad \forall\, l\in\mathcal{L} \qquad \hat{v}_l \le v^{max}, \qquad \forall\, l\in\mathcal{L}$$

(**)

$$\left|\left|\max\{\left|\hat{P}_l^b\right|, \left|\bar{P}_l^b\right|\}\right| + j\max\{\left|\hat{Q}_l^b\right|, \left|\bar{Q}_l^b\right|\}\right|^2 \le v_l I_l^{\max}, \forall l\in\mathcal{L}$$

$$P_l^t \le P_l^{\max} \quad Q_l^t \le Q_l^{\max}, \forall l\in\mathcal{L} \quad \bar{P}_l^t \le P_l^{\max} \quad \bar{Q}_l^t \le Q_l^{\max}, \forall l\in\mathcal{L}$$

Proposed Convex OPF: Augmented Relaxed Optimal Power Flow (AR-OPF)

Lemma I: *If* $(s, S, v, f, \hat{S}, \bar{v}, \bar{f}, \bar{S})$ satisfies (*) and (**), then:

1. $f \leq \bar{f}, v \leq \bar{v}, \hat{P}^t \leq P^t \leq \bar{P}^t$, and $\hat{Q}^t \leq Q^t \leq \bar{Q}^t$
2. If (s, S, v, f) satisfies (*) and $(s, S', v', f', \hat{S}, \bar{v}, \bar{f}', \bar{S}')$ satisfies (*.a), (*.b), (*.d), (*.f), (**) with $0 < v' \leq v$, then $\exists\ (\bar{f}, \bar{S})$ such that $\bar{f} \leq \bar{f}', \bar{P} \leq \bar{P}', \bar{Q} \leq \bar{Q}'$.

Lemma I implies that $\hat{P}^t$ and $\hat{Q}^t$ represent lower bounds on P^t and Q^t, respectively (obtained from the DistFlow equations) and that $\bar{S}$, $\bar{f}$, and $\bar{v}$ are upper bounds on S, f and v, respectively.

Sufficient conditions for exactness

- For **ensuring** the **exactness** of the **relaxation** we have derived **5** (*C1-C5*) **sufficient** conditions
- These **condition** are **mild** and **hold** for **real** distribution **networks** (as we show next)
- They **hold** when the **lines parameters** (z_l and b_l) are **sufficiently small** (as for power distribution grids)

Condition C1:

$$\|\mathbf{H}^{\mathrm{T}}\mathbf{M}\| < 1$$

Condition C2:

$$\|\mathbf{E}\| < 1$$

Condition C3: there exists an $\eta_5 < 0.5$ such that

$$\mathbf{DE} \leq \eta_5 \mathbf{D}$$

Condition C4: there exists an $\eta_1 < 0.5$ such that

$$(\mathbf{H}\mathrm{diag}(r)\ \mathbf{E}) \circ \mathbf{H} \leq \eta_1 \mathbf{H}\mathrm{diag}(r)$$

Condition C5: there exists an $\eta_2 < 0.5$ such that

$$(\mathbf{H}\mathrm{diag}(r)\ \mathbf{EE}) \leq \eta_2 \mathbf{H}\mathrm{diag}(r)\ \mathbf{E}$$

Nick, Cherkaoui, Le Boudec and Paolone 2017 "An Exact Convex Formulation of Optimal Power Flow in Radial Distribution Networks Including Transverse Components." *arXiv preprint arXiv:1605.01964* (2017).

Theorem I: Under conditions C1-C5:

1) For every **feasible solution** of **AR-OPF** there **exists** a **feasible solution** of **O-OPF** with the same power injection vector s.

2) Every **optimal solution** of **AR-OPF** satisfies the original load flow equations, and **is** thus an **optimal solution** of **A-OPF**.

→ Relaxation is exact at the optimal – point 2)

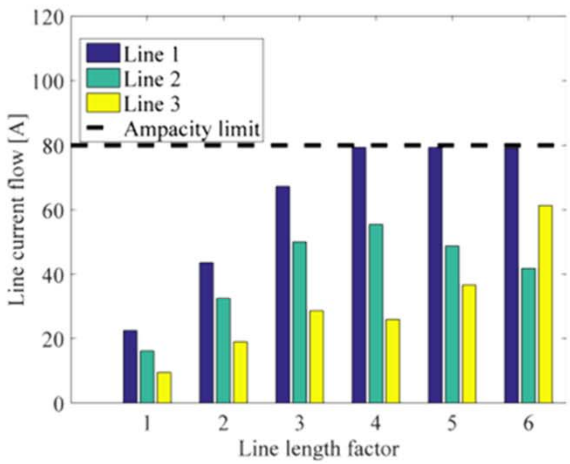
→ Relaxation is exact at every feasible point, in some sense – point 1)

$\neq$ existing relaxation results

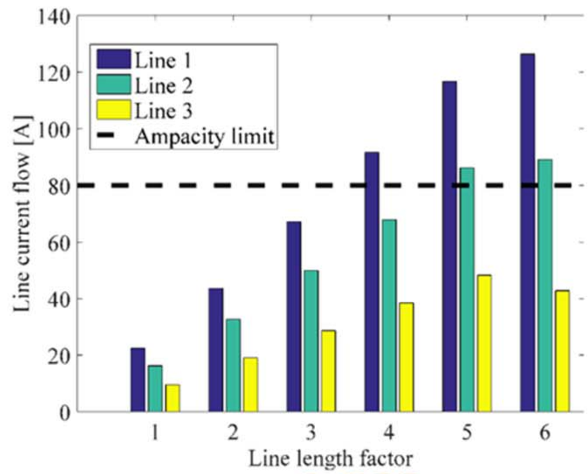
$\Rightarrow$ the set of feasible power injections of AR-OPF is convex

M. Nick, R. Cherkaoui, J.Y, Le Boudec, M. Paolone, "An Exact Convex Formulation of Optimal Power Flow in Radial Distribution Networks Including Transverse Components," arXiv Optimization and Control (math.OC), to appear on IEEE Trans. On Control and Autom.

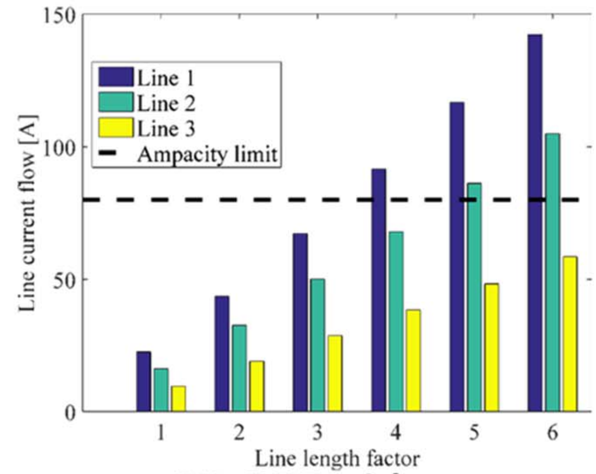
The Importance of Modelling Transverse components



AR-OPF

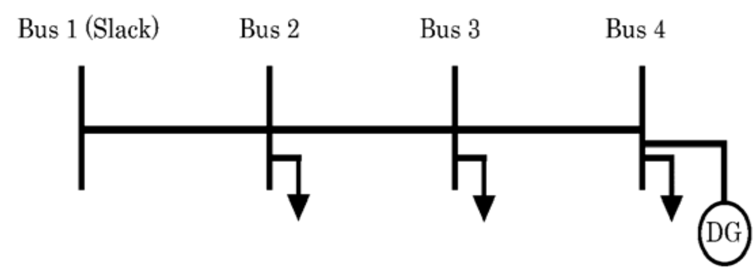


R-OPF



AR-OPF without transverse parameters

Parameter	Value
Line parameters $R\left(\frac{\text{ohm}}{\text{km}}\right), L\left(\frac{\text{mH}}{\text{km}}\right), C\left(\frac{\mu\text{F}}{\text{km}}\right)$, length(km)	(0.193, 0.38, 0.24, 1)
Network rated voltage and base power (kV, MVA)	(24.9, 5)
Power rating (MW) (Storage on bus 3)	1.5
(v^{min}, v^{max}) (p.u.)	$(0.9\times0.9, 1.1\times1.1)$
I_{kl}^{m} (for all 3 lines) (A^2)	120^2
Complex load (3 phase) on bus 2 and 3 (p.u.)	$(-0.21 - j0.126)$, $(-0.252 - j0.1134)$
Energy cost from external grid, cost of active power production/consumption of controllable device at bus 3 ($/MWh)	(150, 50)



Lines are underground coax cables.

Validity of Conditions C1-C5 on two Benchmarks

- IEEE 34-bus test distribution feeder (**weak grid with long overhead lines**)
- CIGRE (EU) benchmark medium-voltage network (**underground cables with high penetration of distributed energy resources)**

Constraints:

- min-max nodal voltage magnitudes limits: 0.95 and 1.05 p.u.
- line ampacities: as listed in the benchmark grid data

Results:

- IEEE 34-bus: first condition that is violated is ***C5*** for a **total injection equal to 1241.8 kW and 755.35 kVAr** respectively. For this operating point, the **nodal voltage-magnitudes reach a maximum value of 1.115 p.u., therefore, far from feasible operating conditions.**
- CIGRE (EU): the first conditions that are violated are ***C3*** and ***C5***. These violations occur at **570% of the base value of DG injection (i.e., 17.55 MW)**. For this operating point, **the maximum value of the nodal voltages reaches 1.105 p.u., again a value far from feasible operating conditions.**

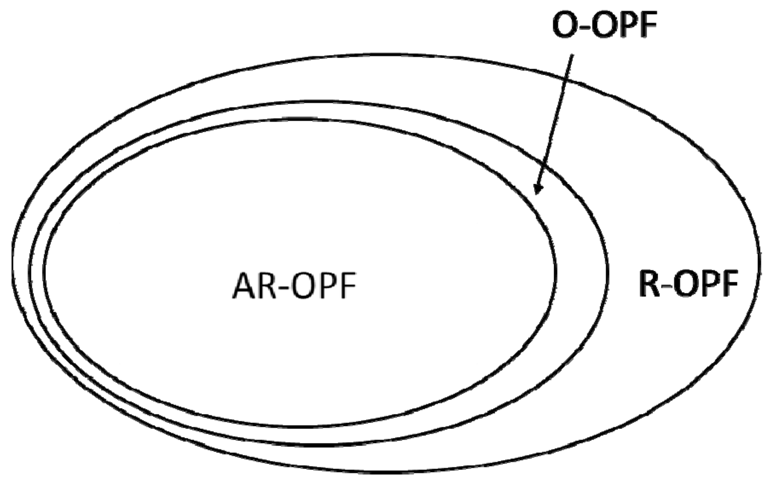
Quantification of the compression of the solution space

Procedure (for node voltage constraints)

- remove ampacity limits
- increase nodal injections up to the point where one augmented voltage constraint is binding

Results

- IEEE 34-bus: Maximum difference between voltage magnitudes and auxiliary ones is equal to 0.0011 p.u.
- CIGRE: the difference is 0.0037 p.u.



Procedure (for ampacity constraints)

- remove voltage constraints
- increase nodal injections up to the point one augmented ampacity constraint is binding

Results

- IEEE 34-bus: Maximum difference between auxiliary current magnitude and true one is equal to 4.34%.
- CIGRE: the difference is 9.88%

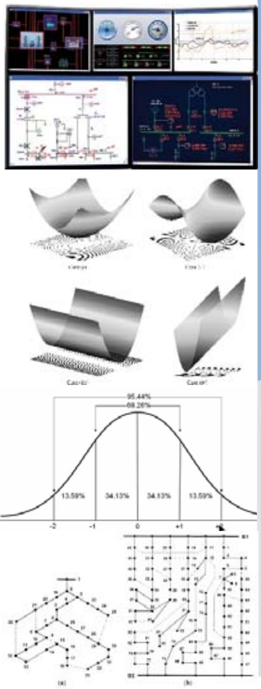
Conclusions

- Existing SOCP relaxation for OPF in radial DNs is not applicable when we model **line flow constraints** and/or **line transverse parameters**.

- Our proposed **AR-OPF**:
  uses the exact two-port $\Pi -$ line model
  models line ampacity limits
  augments the problem constraints to make relaxation exact

- Relaxation differs from previous ones in that it is "**exact everywhere**", not just at optimal.

- Sufficient conditions for feasibility and optimality can be verified ex-ante and are satisfied by benchmark IEEE and Cigré DN grids.



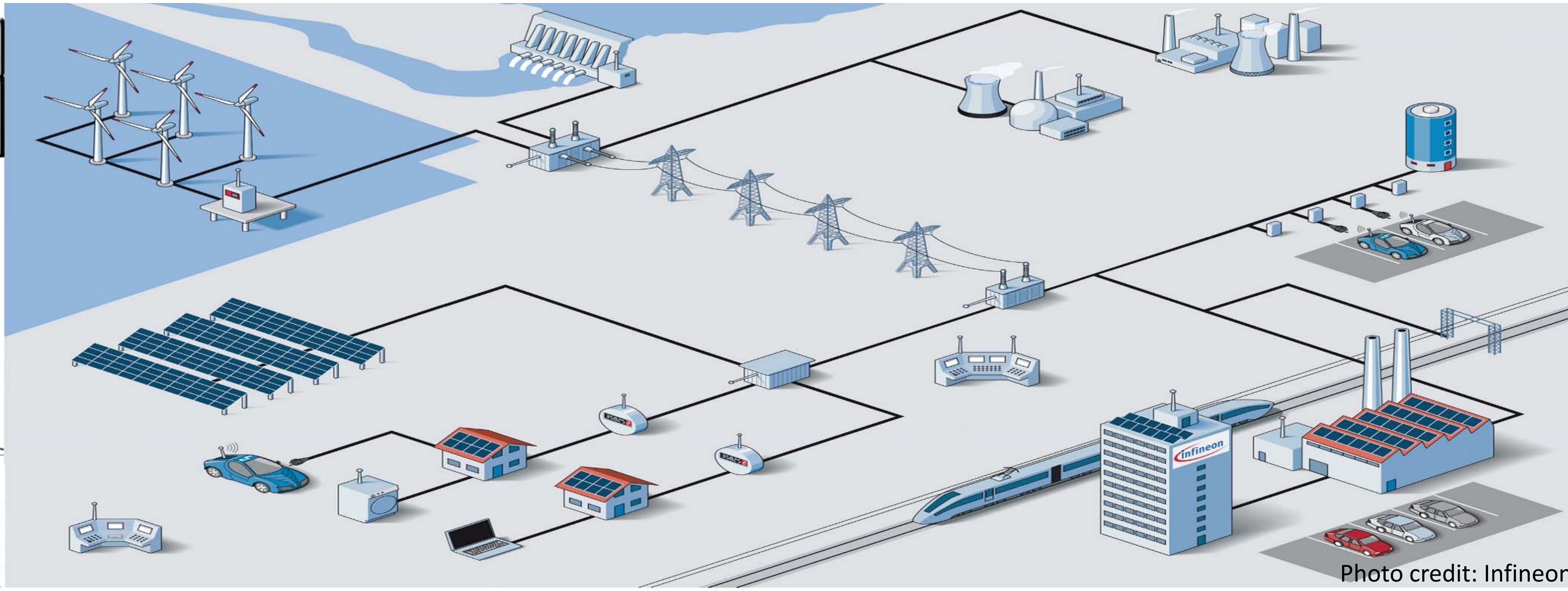


Photo credit: Infineon

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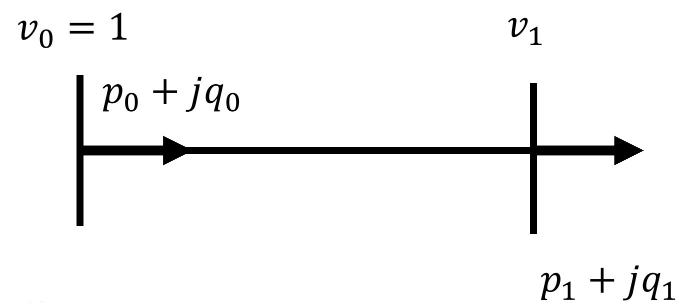
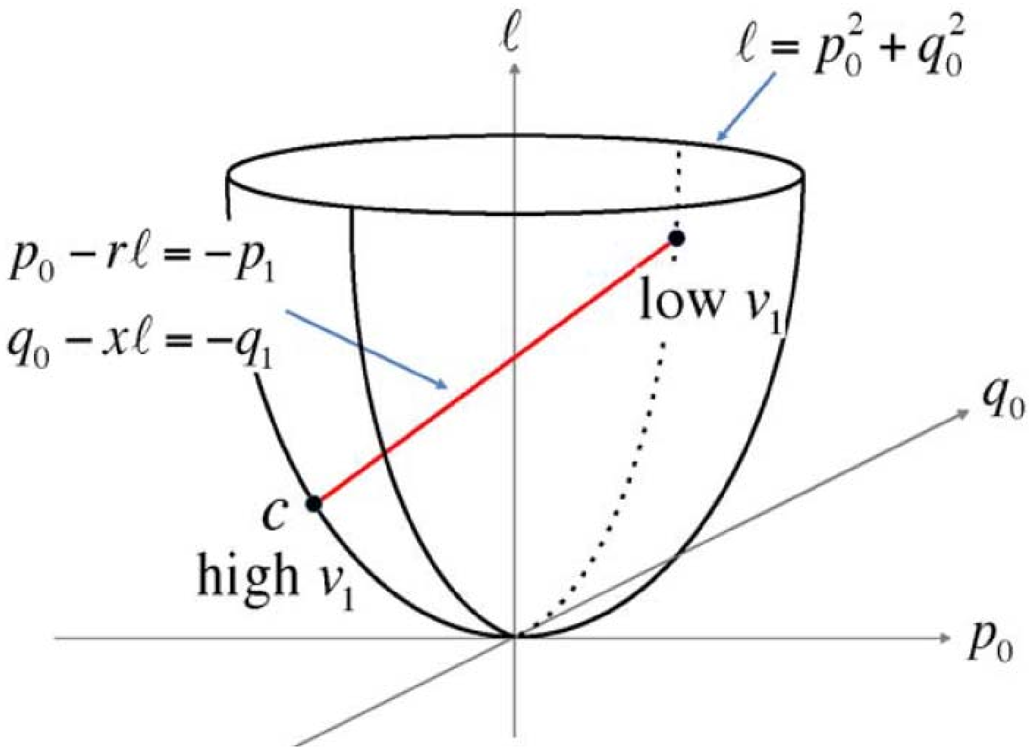
SOCP relaxation



Feasible set of OPF for a two-bus network without any constraint.

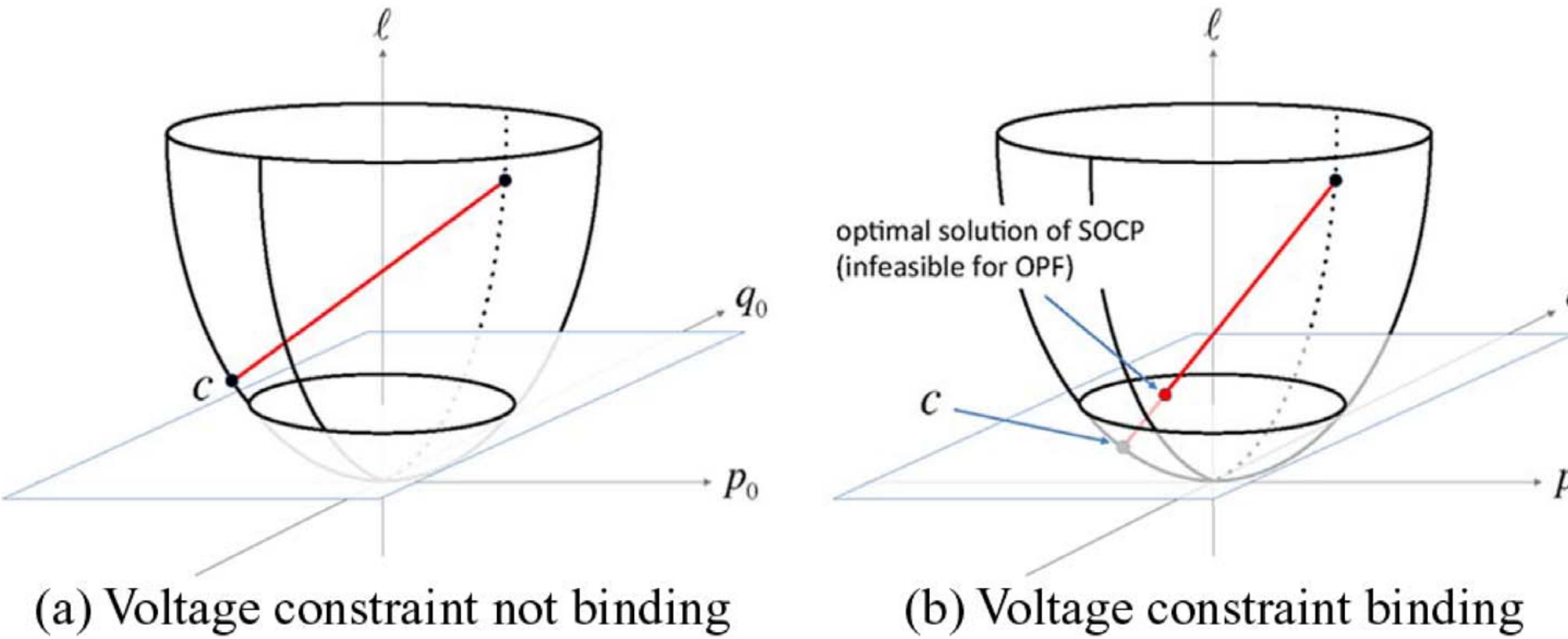


Nomenclature: in this figure we assumed $\ell = f$ (i.e., ℓ is the square of line current magnitude)

Steven Low, “Convex Relaxation of Optimal Power Flow—Part II: Exactness”, IEEE Trans. Power syst.

Impact of static security constraints on the exactness

Voltage upper bound



Steven Low, "Convex Relaxation of Optimal Power Flow—Part II: Exactness", IEEE Trans. Power syst.

Impact of static security constraints on the exactness

Large nodal injection

Inexact solution

$$f_l > \frac{\left| S_l^t + j\frac{v_{\text{up}(l)} b_l}{2} \right|^2}{v_{\text{up}(l)}}$$

$$f_l'' \geq 0$$

$$f_l = \frac{\left| S_l^t + j\frac{v_{\text{up}(l)} b_l}{2} \right|^2}{v_{\text{up}(l)}} + f_l''$$

$$S_l = s_l + \sum_{m \in \mathcal{L}} \left(\mathbf{G}_{l,m} S_m \right) - \frac{i\left(v_{\text{up}(l)} + v_l\right) b_{kl}}{2} + z_l\left(f_l + f_l''\right)$$

$$S_l = \boxed{\left(s_l + z_l\left(f_l''\right)\right)} + \sum_{m \in \mathcal{L}} \left(\mathbf{G}_{l,m} S_m \right) - \frac{i\left(v_{\text{up}(l)} + v_l\right) b_{kl}}{2} + z_l\left(f_l\right)$$

Inexact solution when s_l →

- negative (injection)
- its absolute value is large
  (violation of static security constraints)

f_l'' will get a positive value to satisfy the security constraints

- The solution is might be physically infeasible